KINETIC NATURE OF THE BRITTLE FRACTURE OF SOLID BODIES

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UDC 539.375

INTRODUCTION

Problems of the strength of structures, the calculation of critical loads in machine parts, and the investigation of their fracture have arisen and been solved from the time when the first structures appeared. Brittle cleavage fracture can be regarded as the most dangerous form of failure. In many cases the cracks arise in the surfaces perpendicular to the maximum acting tension, and it is on just these surfaces that separation of the atomic layers takes place [1, 2]. However, it is insufficient to adopt the "critical" point of view even in the case when large-amplitude dynamic stresses act, and a kinetic approach is essential. It is not enough to create the critical level of stress, strain, or any combination of them in the body; we must also take into consideration the time which the situation we have created requires to produce the necessary changes in the material preparatory to its fracture. The majority of investigators are becoming increasingly interested in the time aspects of ridigity, purely theoretically, or from the point of view of determining the properties of materials [3]. Only the first tentative steps have been taken in applying this knowledge to solving concrete problems [4, 5]. The present paper is devoted to a discussion of a series of cases which illustrate the necessity of allowing for the time characteristics of the brittle-fracture process.

§1. In the one-dimensional case we have one of the simplest pictures of a split fracture. It is the one-dimensional picture which is often used as a model for studying aspects of the phenomenon or the properties of materials. Moreover, in a series of real objects the stress fields can be taken as one-dimensional with a good degree of accuracy. Stress waves far from the source in a one-dimensional or layered medium are phenomena of this type, as well as those close to the axis of symmetry of a complicated spatial pattern and also in structures made of rods.

We shall consider the case in which a wave with a triangular profile propagates with no region of growth and without damping, the stress maximum is σ_* and the length is $\lambda = cT$ (T is the duration of the wave; c is its velocity of propagation in the medium). The wave propagates along a rod of length b and is reflected from the free end. At the point of observation x (distance from the free end) and at time $t_1 = (b - x)c^{-1}$ the stress instantaneously assumes the value σ_* and then decreases linearly. The reflected wave arrives at time $t_2 = (b+x)c^{-1}$ with a maximum value of tension σ_*:

$$\sigma = -\sigma_*[1 - (t - t_1)/T]\varepsilon(t - t_1)\varepsilon(T + t_1 - t) + \\ + \sigma_*[1 - (t - t_2)/T]\varepsilon(t - t_2)\varepsilon(T + t_2 - t),$$

where $\varepsilon(t)$ is the unit function. If the point under consideration is situated closer than half a wavelength from the free end $x \leq \lambda/2$, then the waves will overlap. During the time interval $t_3 = T - 2x/c$ the value of the tensile stress of magnitude

$$\sigma = (2x/c)d\sigma/dt = 2x\sigma_*/\lambda \tag{1.1}$$

remains constant, beginning from time t_2, i.e., the dynamic analog occurs of the "static" experimental arrangement of S. N. Zhurkov. This enables us to formulate the inverse experimental–analytical problem of determining the rigidity characteristics of the medium, including the formulation of a special experiment with simple scaling [6, 7].

As an example of this problem we can compare the results of the analysis of split fracture based on the first theory of rigidity (in terms of the greatest tensile stresses)

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 150-157, September-October, 1976. Original article submitted December 21, 1975.

$$\sigma \leqslant \sigma_0 \tag{1.2}$$

and some of its time generalizations, for example,

$$J = \int_0^{\tau_f} \sigma(t)\, dt \leqslant J_0; \tag{1.3}$$

$$\tau = Ae^{-B\sigma(t)} \tag{1.4}$$

with an analysis of the degree of preparedness of the medium to fracture from

$$\Phi = \int_0^{\tau_f} \frac{dt}{\tau} = 1, \tag{1.5}$$

where J_0, A, and B are rigidity constants of the material; in particular, J_0 is the critical value of momentum of the tensile stresses, τ is the durability of the material, and τ_f is the fracture time. The integration is carried out over all the intervals of time in which the stress $\sigma(t)$ is tensile.

If the criterion (1.2) is satisfied, the thickness of the first split fracture δ is equal to half the distance in the wave from its front to the point corresponding to where the stress decreases to the value σ_0. If the maximum amplitude of the wave is several times greater than the breaking point, then multiple splitting occurs [8, 9]:

$$\delta = \sigma_0\lambda/(2\sigma_*); \; n = [\sigma_*/\sigma_0]; \; n\delta \leqslant \lambda/2, \tag{1.6}$$

where n is the number of split fractures, and the square brackets indicate that the whole part of a fraction must be taken. Since fracture occurs as soon as condition (1.2) is satisfied, i.e., at the front of the reflected wave, the rebounding plate "carries off" the leading part of the momentum of the wave which is of length 2δ. Momentum, similar to the initial momentum, is reflected from the new surface, and so the thickness of the subsequent splits is also equal to δ, while the overall width of all the splits does not exceed half the wavelength (1.6).

From the point of view of the time criteria [Eqs. (1.3)-(1.5)] the moment when the reflected tension wave of any amplitude arrives is not critical for a given point, by contrast with the case discussed previously. This part of the wave must prepare the material for fracture, and so at the moment of splitting at a particular point the leading part of the momentum of the tension wave ("critical phase"), having prepared for the fracture, propagates further into the depth of the rod. The rebounding plate "removes" not the beginning, but some central part of the wave momentum, while the remaining part of the momentum on reflection from the new surface adjusts itself to the first "critical phase" of the wave. Acting in the context of the smaller amplitudes of the direct wave, the "critical phase" can prepare the medium for fracture somewhat faster, and there is a tendency for the subsequent split layers to be less thick. In an ideal homogeneous material continuous fracture of a cavitation type can occur to great depths.

Under these conditions [wave with a triangular profile with no region of growth, criterion (1.3)] the momentum of the tensile stresses at points situated at a distance x from the free end of the rod has the form

$$J(x, t) = 2x\sigma_*/\lambda[t - (b + x)/c], \; t_2 < t < t_2 + t_3.$$

We can calculate the time

$$t = J_0\lambda/(2x\sigma_*) + (b + x)/c; \; \delta = x(t_{\min}) = \sqrt{J_0\lambda c/(2\sigma_*)} \tag{1.7}$$

or

$$t = (xc)^{-1}(x^2 + bx + \delta^2); \; \tau_f = (b + 2\delta)/c$$

when the criterion (1.3) is fulfilled at the point in question and the thickness of the first split layer at the minimum point of the curve $\tau_f = \min t(x)$.

The nature of the curves J(x, t) is shown in Fig. 1 as a function of time (the straight continuous lines in the first section) for the case $\sigma_* = 380$ kbar, c = 5 km/sec, T = 7 μsec, b = 40 mm, and $J_0 = 50$ kbar $\cdot\mu$sec. The dashed line corresponds to the critical level of stress momentum. The way in which the line t(x) changes is shown in Fig. 1b with a characteristic minimum at the point of splitting. If we assume that the critical momentum (1.3) is connected with the breaking point for fracture (1.2) by the relation $J_0 = \sigma_0 T_1$, and $T_1/T = \sigma_0/(2\sigma_*)$, then Eqs. (1.6) and (1.7) for δ are identical.

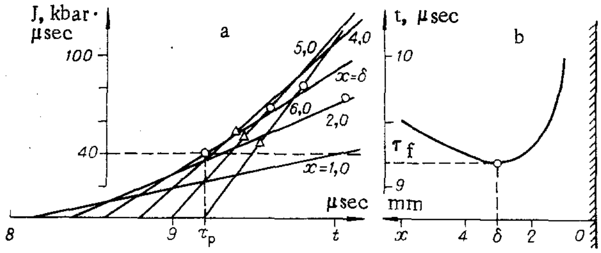

Fig. 1

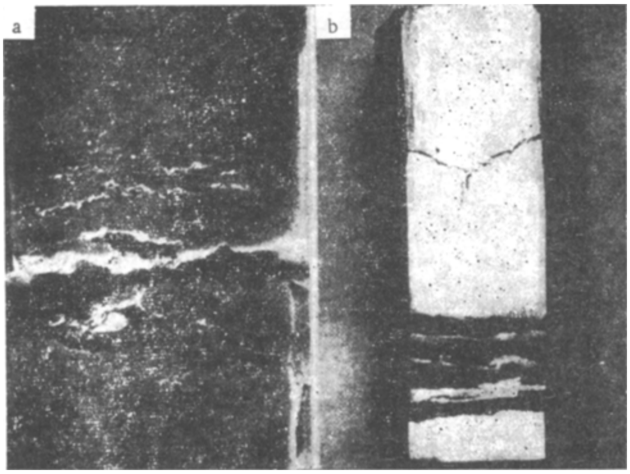

Fig. 2

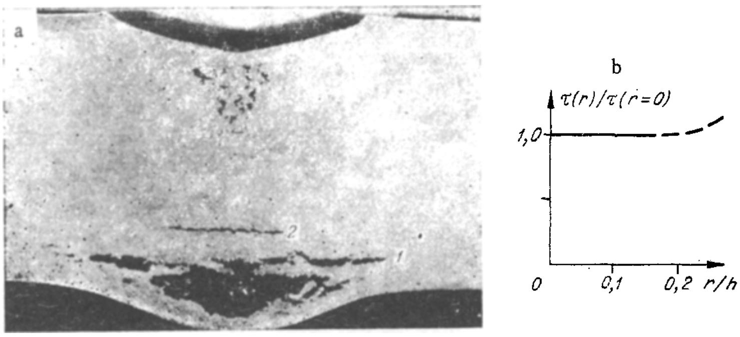

Fig. 3

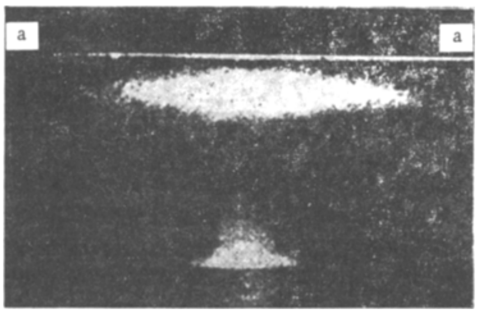

Fig. 4

We shall also consider the question of multiple splitting in more detail. We shall take the coordinate of the first split $x=\delta$ and the moment of the first failure $t=\tau_f$ as the coordinate origin. At all the points $y=x-\delta$, $y>0$, at times $\tau=t-\tau_f$, the momentum of the tensile stresses exceeds the critical momentum:

$$J(y,\ \tau)=J_0+2\sigma_*(\lambda c)^{-1}[c\tau(\delta+y)-y^2]>J_0, \tag{1.8}$$

since at the time under consideration $c\tau>y$. This means that at all the subsequent points $y>0$ the criterion for failure is satisfied, and the conditions exist for continuous fracture and cavitation-type scaling. In this context the following properties of the picture of fracture obtained experimentally are worthy of attention (Fig. 2 [10]). The split surface, the whole transition zone of chaotically fractured "boiling" metal, has a ragged uneven nature (Fig. 2a). After the first split layer there can occur a whole series of very thin split scales (Fig. 2b). In this connection the picture of the first split fracture (1 in Fig. 3a) is very interesting. (Standard 3, samples of $200\times200\times40\ mm^3$, charge on the face of TH (Trotyl–Hexogen) 50/50 of diameter 30 mm and height 30 mm [6]). Here the thickness of the split is difficult to determine, and the ragged chaotic fracture of the material can be seen to some depth beyond the clearly visible first split layer.

At points deeper in the medium the effective part of the stress momentum at the moment of the first splitting acts to the accompaniment of a smaller compression in the direct wave. This means that the process of preparing the medium for fracture is accelerated and that the velocity of the fracture front at these points can be greater than the velocity of sound. In fact, the straight lines (see Fig. 1a) become steeper at the subsequent points. The circles and triangles on these straight lines show the moment when information about the first split arrives (with the velocity of sound), and about splits at a preceding point, for which the corresponding function is also given here. It is characteristic that this information is delayed to the moment when the criterion of rigidity is satisfied and cannot affect the fracture process. As a result of this, the fracture front overtakes the wave front until a part of the effective fracturing momentum of duration

$$\tau_0\simeq J_0T/(2J_*),\ J_*=1/2\cdot\sigma_*T,\ J_0<J_*, \tag{1.9}$$

becomes critical. The causes fracture of the entire split rod and is not reduced in the splitting process.

Information about the velocity of the fracture front can be obtained directly from Fig. 1b or from Eq. (1.7):

$$v=dx/dt=c/(1-\delta^2/x^2),\ x\geqslant\delta. \tag{1.10}$$

Here the point $x(t)$, taken from Eq. (1.7), characterizes the position of the fracture front at time t, the velocity of the fracture front is infinite immediately after the first split for $x=\delta$ and then decreases, approaching the velocity of sound but remaining greater than it. Since Eqs. (1.7)-(1.10) are valid for points at which the interval of time t_3 of constant stress (1.1) is greater than the time of fracture (1.7), i.e., for the points $x_1\leq x\leq x_2$

$$x_1\simeq\delta^2/\lambda;\ x_2\simeq(1/2)\lambda(1-2\delta^2/\lambda^2),$$

then at the points $x\geq x_2$ the tensile stress decreases, and the critical momentum accumulates at a later time. This last fact leads to the velocity of the fracture front decreasing to the velocity of sound more rapidly than indicated by Eq. (1.10).

Some very important concepts about the nature of splitting are outlined in [11], where it is pointed out that the phenomenon is identical with that of cavitation in liquids, on the basis of an analysis of pictures of the fracture of solid bodies obtained experimentally. The moment of cavitation in a liquid [11] is shown in Fig. 4, where we can see a dark zone close to the surface $a-a$, the sharp transition from the undisturbed region to the disturbed region, and the smooth transition from the cavity to the undisturbed zone in the depths of the liquid. These characteristic features are shown in Fig. 1b for the time at which the time criterion of rigidity is satisfied in a solid body. The large extent of the cavity along a surface with comparative uniformity is also interesting.

In this situation the formation of definite pieces, apart from the first split (and not "cavitational dust"), and the termination of the disturbance can be understood, for example, as the result of scatter in the rigidity parameters in a real solid body, and of the earlier formation of the first split fracture (at the minimum of the rigidity constants) as compared with an ideal homogeneous body, which has the effect of freeing a region close to the new free surface from values of the load leading to fracture.

§2. Some interesting qualitative and quantitative properties of the fracture process can be mentioned in the case of the reflection of spherical waves from a free surface. There is rear splitting in plates and subsurface fracture in an underground explosion.

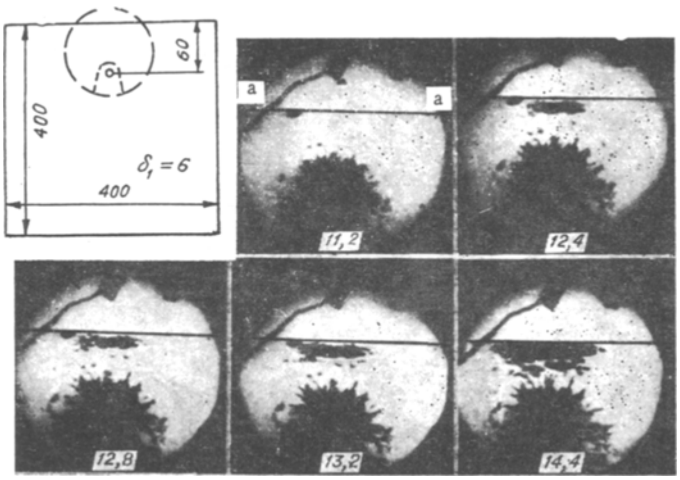

Fig. 5

Some estimates have been made in [12, 13] for splitting fracture in plates based on an analysis of the exact solution of the symmetry axis of the pattern of the phenomenon. In the same papers additional possibilities are discussed for split fracture of the medium to occur when the duration over which the tension phases act is taken into account. The extremal properties of the momentum of tensile stresses in reflected waves are taken into account on this basis, as well as the phase of the tension in the direct transverse wave [13], which plays no small part in the splitting process, by contrast with what was said in [11].

Actually, in this regard, an important property of curves of the vertical component of the stress tensor $\sigma_z(t)$ is the presence in such curves of two phases of the tensile stresses, as well as the variation of the phases themselves and their relative position as a function of the position of the point under consideration. Phases of the tension appear only in the context of unloading in the signal, and the more intense the transition from loading to unloading, the more strongly they appear. The first phase is caused by the presence of the reflected longitudinal wave, while the second results from the action of the direct transverse wave, and has a smaller amplitude but a greater duration than the first.

This variety of possibilities for obtaining tensile stresses σ_z enables us to discuss several variants of the mechanism for initiating a split fracture. For strong disturbances the split fracture is initiated simply by the first reflection in which the tensile values σ_z are large and act long enough for the fracture to develop. In this case thin split layers result. When the disturbance is less strong, the position and location of the fracture can be connected with the extremal properties of this first reflected pulse – the first phase of tension. For still weaker disturbances, when the first phase is insufficient to cause brittle fracture, the second tension phase in the direct shear wave plays an important part. The combined action of these phases can be enough to cause fracture. If we define the location of the fracture as the geometrical location of the points where these phases meet, i.e., the second phase follows immediately after the first, then we can define some spherical surface which recedes into the medium from the rear surface

$$[\eta + 2\gamma^2/(1-\gamma^2)]^2 + \xi^2 = 4\gamma^2(1-\gamma^2)^{-2};\ \eta = z/h;\ \xi = r/h;\ \gamma = v_t/v_l,$$

where h is the thickness of the plate; r, z are cylindrical coordinates; v_t and v_l are the velocities of the transverse and longitudinal waves. The result of such interaction can be seen in Fig. 3a far from the rear of the fracture 2.

A good approximation in the neighborhood of the axis of symmetry can be constructed on the basis of the exact elastic solution on the symmetry axis of the plate and its asymptotic estimates on the wave fronts [6, 13]. The fracture time $\tau(r)/\tau(r=0)$ is shown in Fig. 3b as a function of the radial coordinate, calculated from Eqs. (1.4), (1.5) for a point source. The graph has a characteristic horizontal rectilinear section which indicates that the material in the medium is prepared for fracture simultaneously over the plane surface parallel to the rear side of the plate. This also agrees with the uniformity of the cavity (see Fig. 4) mentioned above. The tendency of the graph to depart from the free boundary can be connected with the return of the split surface back into the depth of the material. The fact that the real effect is not exact, as assumed in the calculations, should lead to a weakening of this characteristic feature.

A similar preparation for fracture (its "delay") also occurs in the case where a cylindrical or spherical wave emerges onto a flat free surface. Figure 5 gives some idea of split fractures in a glass plate [14] of dimensions 400× 400× 6 mm^3, loaded by 100 mg charge of PETN (pentaerithrityl tetranitrate) placed at a distance of

60 mm from one of the boundaries. The experimental arrangement is shown here in a sequence of states of the plate at times given by the numbers in microseconds, while the circle of the field of vision is denoted by the dashed line. It should be noted that the zone of splitting – the dark area close to the surface (the free boundary of the plate is denoted by a–a) – appears abruptly with velocities of 35 and 7 km/sec in the horizontal and vertical directions, respectively, subsequently falls off, and then once again increases abruptly with approximately the same velocities. The facts of retardation and subsequent cessation of the fracture, the extremely high velocities of its front, the structure of the fracture zone (see Fig. 5, the zone consists of many separate small cracks which afterward join up), and the development of the fracture both into the depth of the material as well as toward its free surface (this corresponds to the appearance of secondary split fractures in the fragments which have been split off), can be understood only from the kinetic point of view. It is in this sense that the velocity of the fracture front (and not the velocity of motion of an individual crack), not the apparent velocity, but the real velocity of fracture, can be larger than the velocity of sound in the material, and so to some extent is not characteristic of the material, although it depends on its properties to a large degree, but is rather characteristic of the problem, depending on the geometry and nature of the loading.

Analysis shows that the optimal force and time conditions for fracture are not satisfied on a flat surface parallel to the boundary of the half-space. In the neighborhood of the symmetry axis close to the surface of the half-space ($\xi = rR_0^{-1}$ and $\eta = zR_0^{-1}$ are small quantities such that $1 - \eta > \xi$, where r and z are the distance of the point from the symmetry axis and the surface of the half-space, and R_0 is the characteristic dimension of the problem) the following approximate relation holds for describing the position of this surface:

$$\eta \simeq T/[\gamma(2 - \xi^2)],$$

where T is the duration of the signal. This last relation describes some surface, convex in the direction of the free boundary, with edges receding into the interior of the half-space, which is in agreement with the analogous result for a plate (see Fig. 3b), and has an interesting experimental confirmation [15].

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